EMS-LMS Joint Mathematical Weekend
Celebrating 150 years of the LMS and 25 years of the EMS

University of Birmingham, 18–20 September 2015

ABSTRACTS

Plenary and Special Session Abstracts

Noga Alon

Graphs, vectors and integers

The study of Cayley graphs of finite groups is related to the investigation of pseudo-random graphs and to problems in Combinatorial Number Theory, Geometry and Information Theory. I will discuss this topic, describing the motivation and focusing on several results that illustrate the interplay between Graph Theory, Geometry and Number Theory.

Keith Ball

The probabilistic character of high-dimensional objects

Convex domains play a central role in many areas of analysis and mathematical computer science. During the last 15 years considerable progress has been made in understanding the distribution of mass within such domains in high dimensions. The most telling point is that such domains exhibit characteristics that we normally associate with product probability measures: the joint laws of independent random variables. The talk will survey some of the progress, explain the main open problems and show how our intuition can be badly misleading when we try to think about high-dimensional objects.

József Balogh

On problems of Cameron and Erdos

In 1990 and 1999, Cameron and Erdős proposed several problems in additive combinatorics. They asked about the number of sum-free sets, number of maximal sum-free sets and number of Sidon sets in $[n]$. Additionally they asked for the number of sets in $[n]$ containing no k-term arithmetic progression. In the talk we survey the recent progress on these questions. The talk is based on joint results with Hong Liu, Robert Morris, Wojciech Samotij, Maryam Sharifzadeh and Andrew Treglown.

Franck Barthe

Gaussian kernels also have gaussian minimizers

The title hints at E. Lieb's celebrated result on maximizers of multilinear gaussian kernels acting on products of L_p spaces. In a joint work with P. Wolff, we put forward a similar principle for minimizers when the indices p are less than 1 (possibly negative). This extends and unifies many "reverse" inequalities known in the literature. Monotone transportation maps play a key role in the proof, and the presence of negative coefficients requires new ingredients.

Béla Bollobás

Percolation and random cellular automata

The theory of percolation was founded by Broadbent and Hammersley in 1957: it is the study of random subgraphs of (infinite) lattice-like graphs. E.g. We may take the random induced subgraph $\mathbb{Z}_p^d$ of $\mathbb{Z}^d$ obtained by selecting the sites (vertices) of $\mathbb{Z}^d$ with probability p, independently of each other. Here, as in all 'natural' models of percolation, it is immediate that there is a *critical probability* p_c such that if $p < p_c$ then a.s. $\mathbb{Z}_p^d$ has no infinite component, while for $p > p_c$ it does. (Determining this critical probability is a different matter.)

A cellular automaton, introduced in the 1940s by von Neumann after a suggestion of Ulam, is an interacting particle system. In its simplest form, it starts with a certain configuration, a collection of sites of a grid, after which at each time-step the system updates itself according to some fixed *local* rule: each site goes into a state that depends only on the states of a few nearby sites.

One of the simplest cellular automata is *bootstrap percolation with infection parameter* r, whose study was initiated by Chalupa, Leith and Reich in 1979. This process is an oversimplified model of the spread of an infection on a graph: each site may be *healthy* or *infected*, with an infected site remaining infected for ever, and a healthy site getting infected if it has at least r infected neighbours. The initial configuration is said to *percolate* if every site will be infected at some stage. Disappointingly, if in r-neighbour bootstrap percolation on $\mathbb{Z}^d$ the initially infected sites are chosen with the same probability p, independently of each other then, depending on the pair (d, r), the random configuration either a.s. percolates or a.s. fails to percolate: the critical probability is either 0 or 1. Because of this, bootstrap percolation has been studied mostly on finite graphs.

Recently, with Smith and Uzzell, I introduced a far-reaching generalization of bootstrap percolation on lattices and lattice-like finite graphs. The only assumptions we made about our cellular automaton is that it is homogeneous and monotone. Surprisingly, much can be proved about these very general random cellular automata.

In my talk, aimed at a general audience, I shall review many classical theorems, and say a few words about some of the recent results I have obtained with Balister, Balogh, Duminil-Copin, Morris, Przykucki, Smith and Uzzell. In particular, we have proved that for a large class of 'natural' random cellular automata the critical probability p_c is nontrivial: $0 < p_c < 1$.

Anthony Carbery

Magnitudes of balls in Euclidean spaces – an application of analysis to the theory of enriched categories

The notion of the magnitude of a metric space was introduced by Leinster (and developed by Meckes and Willerton) in connection with the theory of enriched categories. The magnitudes of familiar sets in Euclidean space are only known explicitly in relatively few cases. We study the magnitudes of compact convex sets with nonempty interior in Euclidean spaces of odd dimension, and relate them to the boundary behaviour of solutions to certain naturally associated higher order elliptic boundary value problems in exterior domains. We carry out calculations leading to an algorithm for explicit evaluation of the magnitudes of balls, and this establishes the convex magnitude conjecture of Leinster and Willerton in the special case of balls in dimension three and provide some results in higher dimensions. In addition to PDE considerations, the arguments also involve some combinatorial techniques.
This is joint work with Juan Antonio Barceló.

Timothy Gowers

Interleaved products in highly non-Abelian groups.

The following problem is motivated by a question in cryptography. Let A and B be subsets of the group $SL(2,q)\times SL(2,q)$ with positive density. Let $(a,a') \in A$ and $(b,b') \in B$ be chosen independently and uniformly at random. Must the "interleaved product" $aba'b'$ be almost uniformly distributed, in the sense that the probability that it equals g is almost exactly the same for every single $g \in SL(2,q)$? I shall talk about joint work with Emanuele Viola, in which we prove that the answer is yes. I shall also talk about some generalizations of this result, which have further cryptographic consequences.

Niccolò Guicciardini

Reading the *Principia* with the help of Newton

This talk examines an annotation in Newton's hand found by H. W. Turnbull in David Gregory's papers in the Library of the Royal Society (London). It will be shown that Gregory asked Newton to explain him an incomplete demonstration in the *Principia*. This annotation opens a window on Newton's more hidden mathematical methods deployed in his *magnum opus*. The received view according to which the *Principia* are written in geometric style with no help from calculus techniques must be revised.

Mihyun Kang

Giant component, the k-core, and branching processes

The giant component has been one of the central topics in the theory of random graphs, since the seminal work of Erdős and Rényi five decades ago. By now, there exist extensive results on the birth, size, structure, central and local limit theorems of the giant component in the random graph. A key observation in this line of work is that the emergence of the giant component is closely related to the survival of the Galton-Watson branching process. The k-core of a graph is the maximal subgraph of minimum degree k, which is perhaps the most natural generalisation of the giant component. Pittel, Wormald, and Spencer [JCTB **67** (1996), 111–151] were the first to determine the threshold d_k for the appearance of an extensive k-core. Recently, Coja-Oghlan, Cooley, Skubsch, and the speaker derived a multi-type Galton-Watson branching process that describes precisely how the k-core is "embedded" into the random graph $G(n, p)$ for any $k \geq 3$ and any fixed average degree $d = np > d_k$. In this talk we discuss some classical and recent results on the giant component, k-core, and branching processes.

Benjamin Klopsch

Representation growth of groups

The representation growth of a group G is given by the numbers $R_n(G)$ of irreducible complex representations of G of dimension at most n. In my talk I will survey some of the key results in the subject and discuss the following theorem (joint work with Avni, Onn, Voll): If a 'semisimple' arithmetic group Γ, such as $\mathrm{SL}_n(\mathbb{Z})$, has the Congruence Subgroup Property, then the degree of representation growth of Γ depends only on the absolute root system of the underlying algebraic group.

The theorem relates to a conjecture of Larsen and Lubotzky on the representation growth of irreducible lattices in higher rank semisimple groups. Its proof combines methods from the representation theory of finite groups of Lie type and techniques to study zeta functions associated to compact p-adic Lie groups. Time permitting, I will discuss a more general set-up (recent joint work with Kionke), where we study zeta functions attached to infinite-dimensional representations of a compact p-adic Lie group.

Michael Krivelevich

Mischievous waiters, modest clients, and probabilistic intuition

Waiter-Client games (also called Picker-Chooser games) are a natural type of positional games, defined as follows. For a finite set X, a family F of subsets of X, and a positive integer q (the so called game bias), in each round of the $(1 : q)$ Waiter-Client game (X, F), the first player, called Waiter, offers the second player, called Client, $q + 1$ elements of the board X which have not been offered previously. Client then chooses one of these elements which he claims, and the remaining q elements go back to Waiter. Waiter wins the game if by the time every element of X has been claimed by some player Client has claimed all elements of some subset A from F, otherwise Client is the winner.

In this talk we present several recent results about Waiter-Client games played on the edges of the complete graph K_n on n vertices, comparing them to other game types, such as Maker-Breaker games, and discussing striking similarities to - and differences with - standard models of random graphs.

Martin Liebeck

Some character theory of finite simple groups, with applications

For a finite group G, a character ratio is a ratio of the form $\chi(g)/\chi(1)$ where g is an element of G and χ is an irreducible character of G. These ratios occur natural in a variety of contexts, for example as the eigenvalues of various Markov processes associated with G. In the talk I will survey some old and new results on character ratios, and describe some applications to the theory of growth and random walks on simple groups, and also to representation varieties of various finitely presented groups.

Gunter Malle

Counting characters of finite groups

The development of the character theory of finite groups in the past two decades has been guided by a set of local-global, or counting conjectures, relating the character theory of a finite group to that of its local subgroups. Recently, most of these conjectures have been reduced to (difficult) questions on finite simple groups, thus opening the way to a use of the classification. We will review recent progress on these conjectures.

Marc Noy

Logical limit laws in graph theory

A classical result is that, for every graph property P expressible in first order logic, the probability that a random graph (with the uniform distribution) satisfies P tends either to 0 or to 1. This has been extended in several directions, including the $G(n, p)$ model and random regular graphs.

We study zero-one laws in trees, planar graphs and, more generally, minor-closed classes of graphs. We show that the zero-one law holds for connected planar graphs, even in monadic second order logic (first order logic plus quantification on sets of vertices). For arbitrary planar graphs there is a convergence law, that is, every property has a limit, but not necessarily 0 or 1.

For graphs of fixed positive genus the situation is different. There is a zero-one law for connected graphs in first order logic but not in monadic second order logic. The proofs are based on structural properties satisfied by almost all graphs of fixed genus.

Bob Oliver

Automorphisms of fusion systems of finite groups of Lie type

The fusion system of a finite group G with respect to a Sylow subgroup $S \in \mathrm{Syl}_p(G)$ is the category $\mathcal{F}_S(G)$ whose objects are the subgroups of S, and whose morphisms are the homomorphisms induced by conjugation in G. Set $\mathrm{Out}(S, \mathcal{F}_S(G)) = \mathrm{Aut}(S, \mathcal{F}_S(G))/\mathrm{Aut}_G(S)$, where $\mathrm{Aut}(S, \mathcal{F}_S(G))$ is the group of all automorphisms of S which preserve the fusion in G, and $\mathrm{Aut}_G(S)$ is the subgroup of those automorphisms induced by conjugation in G (by elements of $N_G(S)$). There is a natural homomorphism from $\mathrm{Out}(G)$ to $\mathrm{Out}(S, \mathcal{F}_S(G))$, which in general need be neither injective nor surjective.

In recent work with Carles Broto and Jesper Mller, we looked at the special case where G is a finite group of Lie type. When G is such a group, of universal or adjoint type, and p is the defining characteristic, then with just two exceptions, $\mathrm{Out}(G) \cong \mathrm{Out}(S, \mathcal{F}_S(G))$, and both are isomorphic to a certain outer automorphism group $\mathrm{Out}_{\mathrm{typ}}(\mathcal{L}^c_S(G))$ of the centric linking system of G (a group which also has a topological interpretation). When p is not equal to the defining characteristic and the Sylow p-subgroups of G are nonabelian, there is always some other finite group G^* of Lie type with the same fusion (i.e., $\mathcal{F}_{S^*}(G^*) \cong \mathcal{F}_S(G)$ for $S^* \in \mathrm{Syl}_p(G^*)$), such that the natural homomorphism κ_{G^*} from $\mathrm{Out}(G^*)$ to $\mathrm{Out}_{\mathrm{typ}}(\mathcal{L}^c_{S^*}(G^*))$ is split surjective. In particular, $\mathrm{Out}_{\mathrm{typ}}(\mathcal{L}^c_S(G))$ and $\mathrm{Out}(S, \mathcal{F}_S(G))$ can be described as quotient groups of $\mathrm{Out}(G^*)$. This property (the existence of G^* with the same fusion such that κ_{G^*} is split surjective) comes up in the program of Aschbacher, and in the work of Lynd, when analyzing centralizers of involutions in fusion systems.

Stefanie Petermichl

Optimal control of second order Riesz transforms on multiply-connected Lie groups through processes with jumps.

The precise L^p norms are known for very few classical operators in harmonic analysis. They are usually obtained through the use of a special function of several variables that controls the behaviour of the operator perfectly. In some cases there are striking applications of these optimal L^p norms to regularity problems of certain PDE.

The difficulty increases in an unexpected manner if one considers these operators on discrete sets, such as the integers. The L^p norm of the discrete Hilbert transform on the integers is a famous open question, while the classical question on the unit disk was resolved in the 70s by Pichorides. We answer this question for second order Riesz transforms on products of discrete abelian groups using two different methods, Bellman functions (deterministic) and through stochastic integration. The latter has an extension to compact Lie groups with multiple components and the added benefit to represent the operator applied to a function through conditional expectation of a stochastic integral associated with the function. The underlying processes must have jump components. In the late 70s, Gundy-Varopoulos have done exactly the same for first order Riesz transforms in their beautiful work, but only needing continuous in space stochastic processes.

Cheryl E Praeger

Factorisations of groups

While factorisations of numbers, rings, modules and other algebraic objects reveal important aspects of their structure, the problem of determining when nontrivial factorisations exist, and then finding them, can be difficult (such as in the case of the integer factorization problem underpinning the RSA cryptosystem). These comments apply equally to various kinds of group factorisations, but for groups, the factorisations encountered usually lack the uniqueness properties of these other structures. I will discuss several kinds of group factorisations, and the geometric, algebraic, or combinatorial questions which have led to their study – and indeed what we do, and do not, know about these factorisations. The motivating questions include deciding whether a given subgroup is maximal, or a given transitive graph is a Cayley graph, or a given group is admitted as a flag-transitive automorphism group of a certain point-line geometry.

Sandra Pott

Matrix weights: On the way to the linear bound

In recent years, the attempt to prove sharp bounds for Calderon-Zygmund operators on weighted L^p spaces in terms of the A_p- or A_∞ characteristic of the weight has been an important driving force in Harmonic Analysis. After the work of many authors, this culminated with the proof of the conjectured linear bound for $p = 2$ for all Calderon-Zygmund operators by Tuomas Hytönen in 2010.

Recently, the question of the validity of the linear bound for all Calderon-Zygmund operators in the matrix-weighted setting has attracted some interest. In the talk, I want to present the reduction of this question to the case of Haar multipliers and dyadic paraproducts. I also want to talk about the remaining obstacles, some of which have very recently been resolved, and discuss some consequences of the conjectured linear bound for weighted commutators in the scalar setting.

This is joint work with Andrei Stoica.

Wojciech Samotij

Counting H-free graphs

In this talk, we shall survey a large body of enumerative results in the context of *H-free graphs*, that is, graphs not containing a copy of a fixed graph H as a subgraph. In particular, we shall show how the "hypergraph containers" theorem of Balogh, Morris, and the speaker, proved independently by Saxton and Thomason, allows one to derive (for each H) an approximate structural description of a typical (random) H-free graph with a given number of vertices and edges. In several interesting cases, such as when H is a clique, these approximate structural descriptions may be made precise. We shall also mention several challenging open questions.

Mathias Schacht

Turán-type problems for quasirandom hypergraphs

Extremal problems for hypergraphs concern the maximum density of large hypergraphs H that do not contain a copy of a given hypergraph F. Estimating the so-called Turán-densities is a central problem in combinatorics. However, despite a lot of effort precise estimates are know for only known for very few hypergraphs F.

We consider a variation of the problem, where the large hypergraphs H satisfy additional quasirandom conditions on its edge distribution. We present recent progress based on joint work with Reiher and Rödl for 3-uniform hypergraphs. In particular, we established a computer-free proof of a recent result of Glebov, Král' and Volec on the Turán-density of so-called weakly quasirandom hypergraphs not containing the 3-uniform hypergraph with three edges on four vertices.

Aner Shalev

Groups in interaction

Modern Group Theory gained a lot from its interaction with many other fields of mathematics. These include Probability, Representation Theory, Algebraic Geometry and Number Theory. In my talk I will demonstrate some of these interactions, with emphasis on recent advances in Finite Simple Groups, Lie Groups and other infinite groups.

Benny Sudakov

Two short stories in extremal combinatorics

In this talk we present variants of two classical extremal problems: estimating Ramsey numbers for cliques and Turán numbers for complete bipartite graphs. Our results, which is joint work with Conlon and Fox, are obtained by basic probabilistic arguments and answer questions of Bollobás, Erdős, Foucaud, Hajnal, Krivelevich and Perarnau.

Donna Testerman

Regular unipotent elements and SL_2 subgroups in simple linear algebraic groups

Each simple linear algebraic group defined over an algebraically closed field has a unique conjugacy class of elements (the so-called regular unipotent elements) which is dense in the variety of unipotent elements. This class plays an important role in the theory of these groups.

In this talk, we first trace some of the history of the study of closed subgroups containing regular unipotent elements, concentrating on SL_2-type subgroups, both finite and infinite. We also try to point out some of the interesting applications of these rank one subgroups to the solution of various problems. Finally, we discuss some recent joint work with Tim Burness on finite PSL_2 subgroups containing regular unipotent elements in the exceptional type simple algebraic groups.

Christoph Thiele

Entangled singular integrals

We are interested in L^p bounds for the triangular Hilbert transform

$$\int\int f(x,y)g(y,z)h(z,x)\frac{1}{x+y+z}\,dxdydz\ .$$

This is called an entangled singular integral because of the Loomis-Whitney type structure of the arguments of the three functions. The word "entangled" was used before in the context of similar multilinear forms with a four-cycle of functions. Bounds for the triangular Hilbert transform would reprove a number of celebrated results in harmonic analysis such as Carleson's theorem on convergence of Fourier series as well as bounds for the bilinear Hilbert transform. Recent progress in joint work with Vjekoslav Kovac and Pavel Zorin-Kranich proves such bounds for a dyadic model operator under the assumption that one of the three functions f, g, h satisfies additional structural assumptions.

Luis Vegas

An isoperimetric-type inequality for Dirac Hamiltonians with electrostatic shell interactions

I shall present some recent work in collaboration with N. Arrizabalaga and A. Mas about the spectral properties of the coupling $H + aV$, where H is the massive free Dirac operator in $3d$, and aV is an electrostatic shell potential (which depends on the coupling real parameter a) located on the boundary of a smooth domain. Our main result is an isoperimetric-type inequality for the admissible range of a's for which the coupling $H + aV$ generates pure point spectrum in (m, m). That the ball is the unique optimizer of this inequality is also shown. Regarding some ingredients of the proof, we make use of the Birman-Schwinger principle adapted to our setting in order to prove first some monotonicity property of the admissible a's, and then a sharp constant of a quadratic form inequality, from which the isoperimetric-type inequality is derived.

Julia Wolf

Current challenges in quadratic Fourier analysis

Quadratic Fourier analysis has its origins in quantitative approaches to Szemerédi's theorem, but has also found numerous other applications in combinatorics, analysis and theoretical computer science. Just like classical Fourier analysis, quadratic Fourier analysis allows us to decompose a function into a structured and a random-looking part. However, in the latter case the notion of "random-looking" is more refined, and consequently the structured part is no longer linear but quadratic in nature. This talk will survey the state-of-the-art in the subject and highlight some recent applications.

Postdoctoral Session Abstracts

Jonas Azzam — **Rectifiability of harmonic measure**

The local F. and M. Riesz theorem of Bishop and Jones says that, for a simply connected planar domain, harmonic measure is absolutely continuous with respect to 1-dimensional Hausdorff measure on the intersection of the boundary with any rectifiable curve. We will survey some recent generalizations of this theorem and its converse to higher dimensions. This is based on joint work with Steve Hofmann, José María Martell, Svitlana Mayboroda, Mihalis Mourgoglou, Xavier Tolsa, and Alexander Volberg.

Ben Barber — **Edge-decompositions of graphs**

An *F-decomposition* of a graph G is a partition of $E(G)$ into copies of F. Determining whether a graph has an F-decomposition is NP-complete, but it is much easier to find *fractional* F-decompositions. I'll describe the connection between these ideas and how it can be exploited to attack Nash-Williams' conjecture that every large graph G on n vertices with all degrees even, $e(G)$ divisible by 3 and $\delta(G) \geq 3n/4$ can be decomposed into triangles.

Alonso Castillo-Ramirez — **Finite semigroups of cellular automata**

Since first introduced by John von Neumann, the notion of cellular automaton has grown into a key concept of computer science, physics and theoretical biology. In its classical setting, a cellular automaton is a transformation of the set of all configurations of a regular grid such that the image of any particular cell of the grid only depends on the state of a finite number of its neighbours. The most famous example of a cellular automaton is Conway's Game of Life, whose setting is a two-dimensional infinite grid.

In this talk, we review some recent group theoretic results in the theory of cellular automata (such as Bartholdis characterisation of amenable groups), and then we embark in the study of cellular automata over finite grids from the point of view of semigroup theory. This is joint work with Maximilien Gadouleau.

Endre Csóka — **Limits of some combinatorial problems**

We show some new examples how limit theory of combinatorial structures can help solving combinatorial problems. First, we generalize the Manickam–Miklós–Singhi Conjecture, using limit theory. Then we introduce two limit problems of Alpern's Caching Game, which are good approximations of the finite game when some parameters tend to infinity. With the use of these limit problems, we show a surprising result which disproves some conjectures about the finite problem.

Taryn C Flock

Regularity of the Brascamp-Lieb constant

The Brascamp-Lieb inequality generalizes many important inequalities in analysis, including the Hölder, Loomis-Whitney, and Young convolution inequalities. Sharp constants for such inequalities have a long history and have only been determined in a few cases. We investigate the stability and regularity of the sharp constant as a function of the implicit parameters. The focus of the talk will be a local-boundedness result with implications for certain nonlinear generalizations arising in PDE. This is joint work with Jonathan Bennett, Neal Bez, and Sanghyuk Lee.

Richard Gratwick

The limits of partial regularity in the calculus of variations

Minimizers of variational problems can typically be proved to be regular under suitable smoothness and growth conditions on the integrand. We shall examine what happens in one-dimensional situations where these assumptions fail. In particular we shall consider the applicability of Tonelli's partial regularity theorem. When it does not apply, we shall see some highly degenerate behaviour of minimizers, but yet see that some extra information is still available.

Jie Han

Beyond the Hilton-Milner Theorem

For $k \geq 3$ and $n \geq 2k+1$, we determine the maximum size of a non-trivial intersecting family of k-sets on $[n]$ which is not a subfamily of the so-called Hilton-Milner family. This extends a result of Hilton-Milner from 1967.
This is a joint work with Yoshiharu Kohayakawa.

Felix Joos

The Erdős-Pósa property of rooted minors

A family $\mathcal{H}$ of graphs is said to have the *Erdős-Pósa property* if there is a function $f : \mathbb{N} \to \mathbb{N}$ so that any graph contains k disjoint subgraphs that are isomorphic to graphs in $\mathcal{H}$, or if it contains a vertex set of size at most $f(k)$ meeting all such subgraphs. In the famous graph minors series Robertson and Seymour prove that the family of all H-expansions (all graphs that contain H as a minor) has the Erdős-Pósa property if and only if H is planar. We extend this result to rooted H-expansions and describe exactly for which rooted graphs H the family of rooted H-expansions has the Erdős-Pósa property.
This is joint work with Henning Bruhn and Oliver Schaudt.

Polyxeni Lamprou

Slices for parabolic subalgebras of a semisimple Lie algebra.

Let $\mathfrak{g}$ be a semisimple algebra, $S(\mathfrak{g})$ its symmetric algebra and $Y(\mathfrak{g})$ the Poisson centre of $S(\mathfrak{g})$. It is well-known that $Y(\mathfrak{g})$ is a polynomial algebra. The Slice theorem due to Kostant says that, if $\{x, h, y\}$ is a principal $\mathfrak{sl}_2$ triple in $\mathfrak{g}$, then restriction of functions gives an isomorphism

$$Y(\mathfrak{g}) \xrightarrow{\sim} R[y + \mathfrak{g}^x],$$

where $\mathfrak{g}^x$ is the centralizer of x in $\mathfrak{g}$ and $R[y + \mathfrak{g}^x]$ denotes the ring of regular functions on $y + \mathfrak{g}^x$.
Let now $\mathfrak{p}$ be a (truncation of a) parabolic subalgebra of $\mathfrak{g}$. By a result of Fauquant-Millet and Joseph, it is known that $Y(\mathfrak{p})$ is polynomial in most cases-for example when $\mathfrak{g}$ consists of components of type A and C. I will present an analogue of Kostant's slice theorem for $Y(\mathfrak{p})$. The role of the $\mathfrak{sl}_2$-triple will be played by an *adapted pair*. Moreover, an adapted pair may be used to prove (or disprove) polynomiality of $Y(\mathfrak{p})$ in the cases that it is not known.

Eoin Long

Set families with a forbidden pattern

A *balanced pattern* of order $2k$ is an element $P \in \{+, -\}^{2k}$, where both signs appear k times. Two sets $A, B \subset [n]$ form a P-pattern, which we denote by $\mathrm{pat}(A, B) = P$, if $A \triangle B = \{j_1, \ldots, j_{2k}\}$, with $1 \leq j_1 < \cdots < j_{2k} \leq n$ and $\{i \in [2k] : P_i = +\} = \{i \in [2k] : j_i \in A \setminus B\}$. We say $\mathcal{A} \subset \mathcal{P}[n]$ is P-free if $\mathrm{pat}(A, B) \neq P$ for all $A, B \in \mathcal{A}$. We consider the following natural extremal question: how large can a family $\mathcal{A} \subset \mathcal{P}[n]$ be if $\mathcal{A}$ is P-free?
We prove a number of density results. In particular, we show that if P is a k-balanced pattern with $k < c \log \log n$ then $|\mathcal{A}| = o(2^n)$, where $c > 0$ is a fixed constant. We then give stronger bounds in the cases when (i) P consists of k plus signs, followed by k minus signs and (ii) P consists of alternating signs. In both cases, if $k = o(\sqrt{n})$ then $|\mathcal{A}| = o(2^n)$. In the case of (i), this is tight. Joint with Ilan Karpas.

Theodore Molla

Triangle factors in graphs, directed graphs and weighted graphs

In 1963 Corrádi and Hajnal proved that if G is a graph on n vertices, n is divisible by 3 and $\delta(G) \geq 2n/3$, then G contains a triangle factor, i.e. a collection of $n/3$ vertex disjoint copies of K_3. Since every graph G on n vertices with $\alpha(G) > n/3$ does not have $n/3$ vertex disjoint triangles, the theorem is sharp. In this talk, we will describe several related theorems for graphs, directed graphs and weighted graphs. For example, we will discuss the following recent result: For every $\varepsilon > 0$ there exists $\gamma > 0$ such that if G is a graph on n vertices, $\delta(G) \geq (1/2 + \varepsilon)n$ and $\alpha(G) \leq \gamma n$, then G has a triangle factor when n is sufficiently large and divisible by 3.

This is joint work with József Balogh and Maryam Sharifzadeh

Luke Morgan

Locally semiprimitive arc-transitive graphs

An arc-transitive graph is locally semiprimitive if the stabiliser of each vertex induces a semiprimitive group on the neighbourhood of that vertex. Potočnik, Spiga and Verret conjecture that there is a function bounding the order of stabilisers of vertices in such graphs in terms of the valency of the graph. The theory of semiprimitive permutation groups is still in its infancy, and progress on the conjecture has been hampered by this. I will present some recent results which aim to rectify this situation, and give an overview of the impact on the above mentioned conjecture.

Jason Semeraro

Fusion systems over p-groups with an abelian subgroup at index p

We discuss joint work with David Craven and Bob Oliver completing the classification of saturated fusion over p-groups with an abelian subgroup of index p. A key aspect of our proof is the determination of certain indecomposable modules for groups with a Sylow p-subgroup of order p. We obtain unboundedly many new exotic fusion systems.

Bartosz Walczak

Dimension and structure of partially ordered sets

The dimension of a poset P is the minimum number of linear extensions whose intersection gives rise to P. This parameter plays a similar role for posets as the chromatic number does for graphs. Recent research has revealed unexpected connections between the dimension of a poset and the structure of its cover graph (Hasse diagram). I will present some results in this vein, providing bounds for the dimension of posets with cover graphs with bounded tree-width/excluded minor/other structural restrictions. This is a joint work with Gwenaël Joret, Piotr Micek, Kevin Milans, William T. Trotter, Ruidong Wang and Veit Wiechert.

Lutz Warnke

The phase transition in Achlioptas processes

In the *Erdős–Rényi random graph process*, starting from an empty graph, in each step a new random edge is added to the evolving graph. One of its most interesting features is the 'percolation phase transition': as the ratio of the number of edges to vertices increases past a certain critical density, the global structure changes radically, from only small components to a single giant component plus small ones.

In this talk we consider *Achlioptas processes*, which have become a key example for random graph processes with dependencies between the edges. Starting from an empty graph these proceed as follows: in each step *two* potential edges are chosen uniformly at random, and using some rule *one* of them is selected and added to the evolving graph.

We shall prove that, for a large class of widely studied rules (so-called bounded-size rules), the percolation phase transition is qualitatively comparable to the classical Erdős–Rényi process. For example, assuming $\varepsilon^3 n \to \infty$ and $\varepsilon \to 0$ as $n \to \infty$, the size of the largest component after step $t_c n \pm \varepsilon n$ whp satisfies $L_1(t_c n - \varepsilon n) \sim C\varepsilon^{-2}\log(\varepsilon^3 n)$ and $L_1(t_c n + \varepsilon n) \sim c\varepsilon n$, where $t_c, C, c > 0$ are rule-dependent constants (in the Erdős–Rényi process we have $t_c = C = 1/2$ and $c = 4$).

Based on joint work with Oliver Riordan.

Pavel Zorin-Kranich

Ajtai-Szemerédi theorem over non-commutative groups

The Ajtai-Szemerédi theorem tells that every subset of Z^2 with positive upper density contains "corners", that is, sets of the form $\{a, a+(0,1)b, a+(1,0)b\}$, $b > 0$. A suitably formulated version of this theorem holds over arbitrary amenable groups. Peculiarly, the triangle removal lemma only seems to imply this version in the presence of a Følner sequence satisfying the Shulman condition, and ergodic theory has to be used in general. Moreover, the corners turn out to be abundant in a number of ways, which I will describe in the talk. Joint work with Q. Chu, V. Bergelson, C. Christopherson, and D. Robertson